Introduction

This study is an exploratory systems-risk analysis examining whether long-duration real-return dynamics exhibit threshold-sensitive resilience behaviour under depletion and volatility stress.

The main finding is an early warning geometry suggesting the need for adaptive regeneration from analysis of time series financial data from 1928 to 2025. The data represents annual returns nominatively earned on major financial asset classes, on an annual basis, inclusive of both cash payout during the year and the price appreciation, based on the value of $100 invested at start of 1928 in real terms.

Objectives

The major objective is to determine current stock market health vs historical health.

The idea is to analyse the data for a scalar growth and decay algorithm that the financial situation most resembles. The hypothesis explored here is that portions of the current market environment may exhibit characteristics of a mature, highly financialised system in which asset appreciation can become increasingly decoupled from broad-based regenerative economic growth.

The input data comprises year x S&P 500, US Small cap, 3-month T.Bill, US T. Bond (10-year) , Baa Corporate Bond, Real Estate, and Gold from 1928 to 2025.

The output is to determine growth requirements, depletion pressure, and stochastic forcing volatility (Δr, Δc, $\Delta\sigma$) needed to reach a safe region.

Data Analysis

The start of the series was the onset of the Great Depression, which explains the steep downward slope. The steep growth line for the recent four years is explained by strong earnings growth in corporate America, particularly in technology and AI sectors, alongside favorable monetary policies such as interest rate cuts. Additionally, a rebound in investor sentiment following the easing of trade tensions and tariffs contributed significantly to the market's upward trajectory.

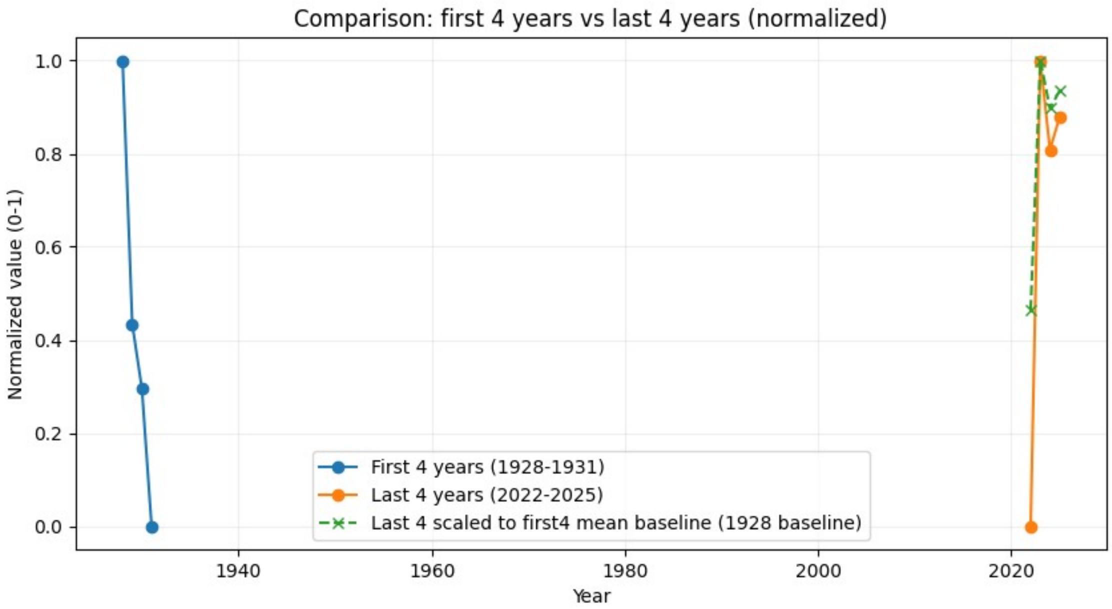


Figure 1: Comparison of first 4 years (1928 – 1931) and last 4 years (2022-2025)

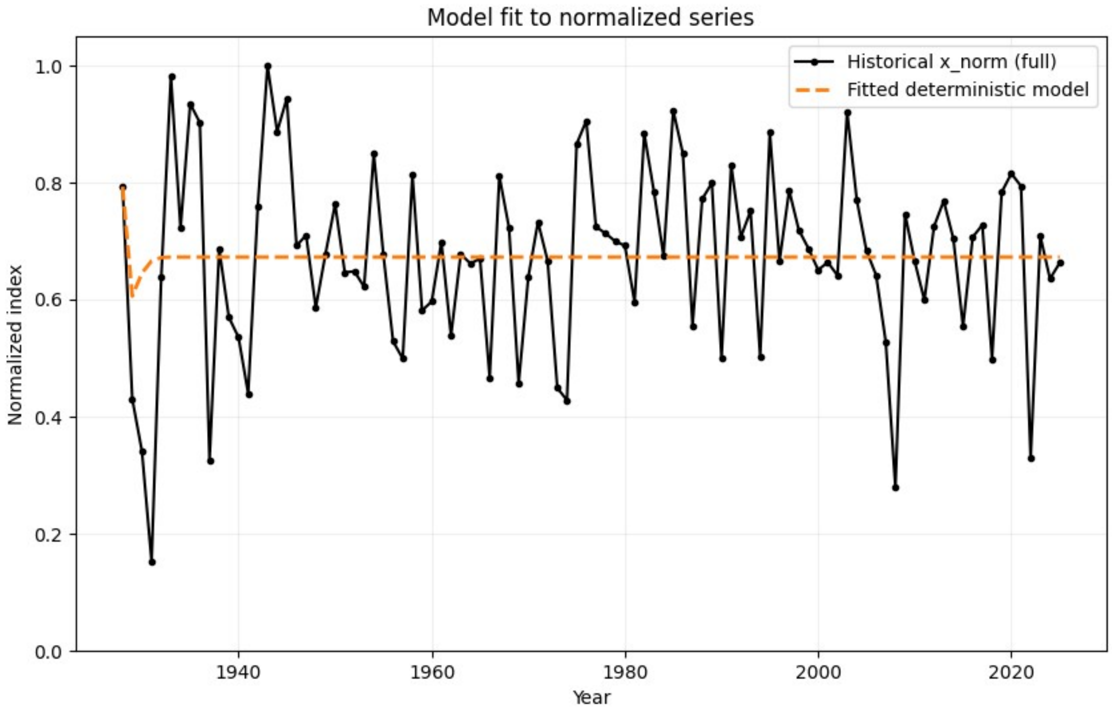


Figure 2: Model fit to a combined normalized index for the time series

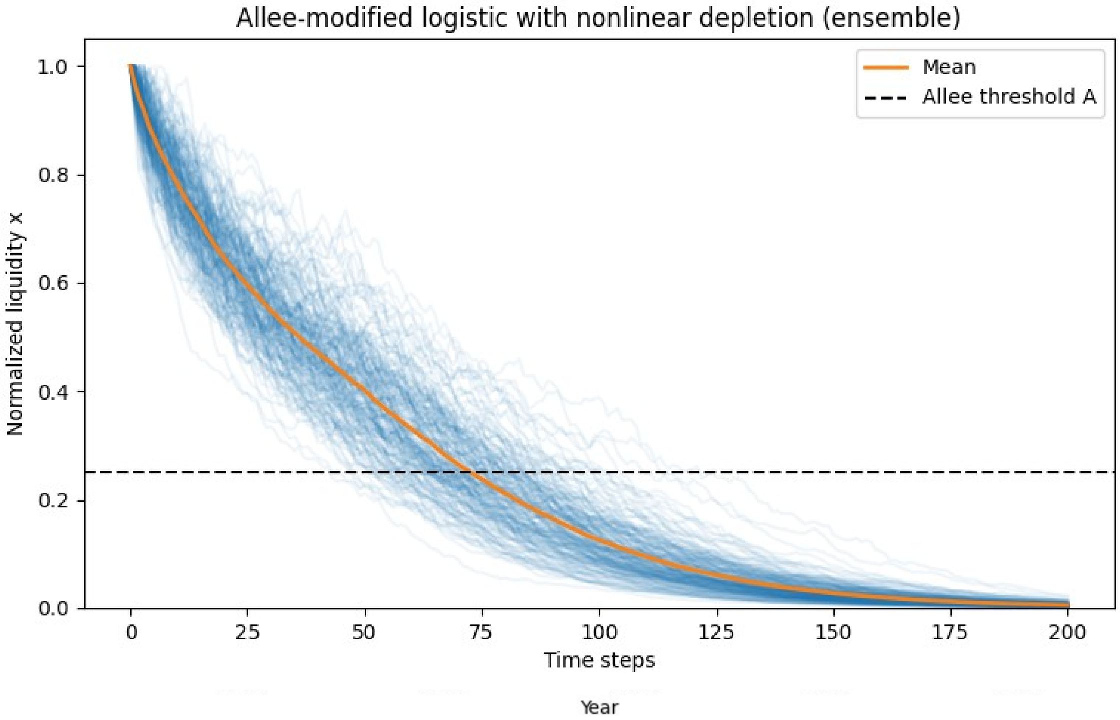


Figure 3: Allee modified logistics model application

The Allee modified logistic model incorporates the Allee effect into the traditional logistic growth model, accounting for positive density dependence at low population sizes. In this context, non-linear depletion refers to the way resources are consumed, depletion, affecting economic growth rates differently depending on population density.

Under the assumptions and scaling of this model, the fitted system sits inside a high-depletion / low-resilience regime where simulated trajectories overwhelmingly drift below the model survivability threshold.

Applied ensemble modeling produced the following results:

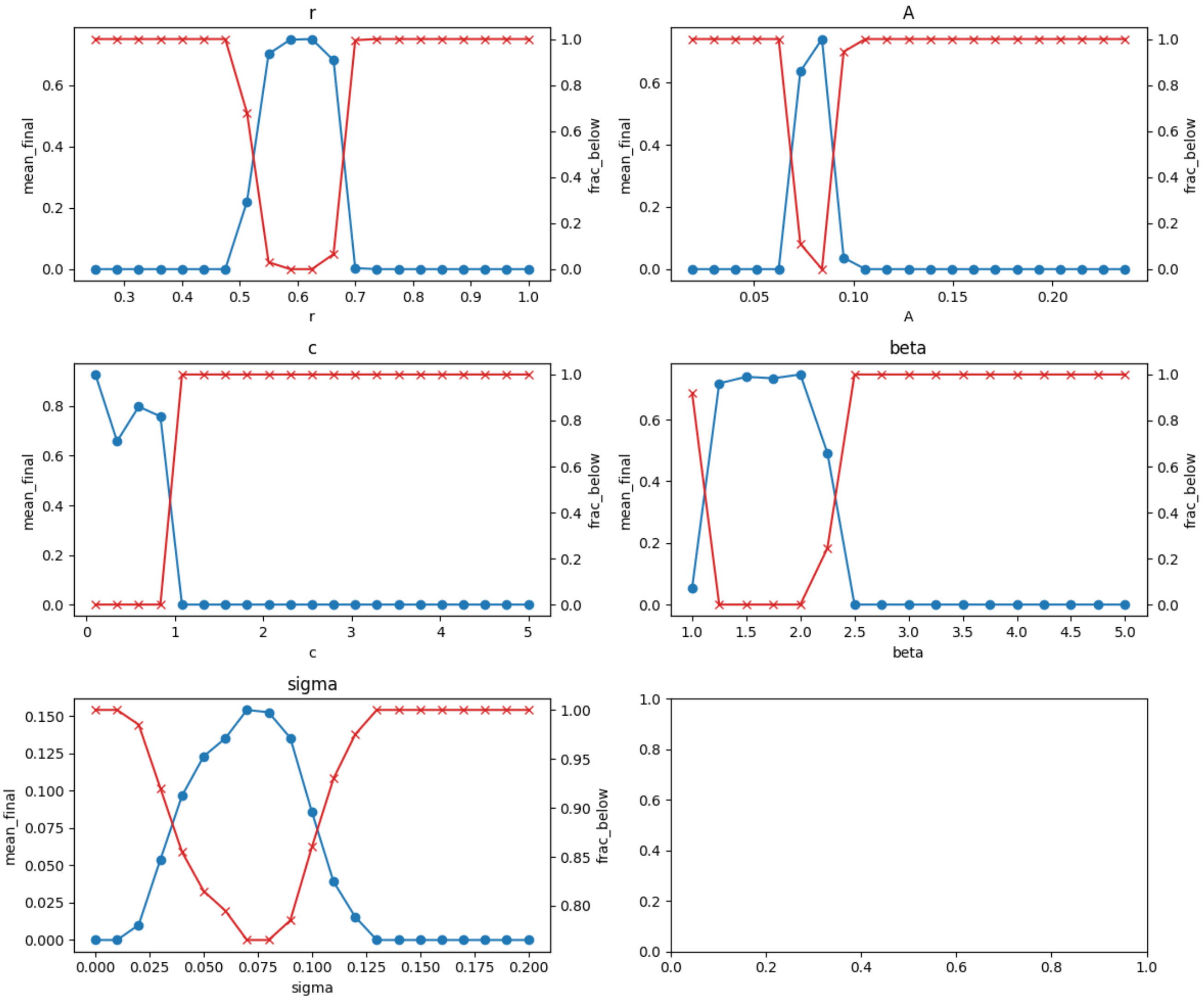


Figure 4: Parameter scanning of stepwise ensemble fitting

Ensemble modeling is applied to r (growth), A (Allee threshold), c (depletion co-efficient) and beta (non-linear exponent) and sigma (noise or volatility from external factors)

General readout conventions:

- Blue line = ensemble mean final x (higher = more resilient).
- Red/orange line (right axis) = fraction of runs ending below the Allee threshold (collapse probability).
- Median time-to-collapse (not shown/NaN → inf) indicates how fast collapse happens when it does.

Panel-by-panel interpretation:

1) r (intrinsic growth)

- Pattern: mean_final rises roughly linearly with r; collapse probability ≈ 0 across the scanned r.
- Implication: increasing r reliably improves long-run outcomes; under current c, A, beta, noise, system is robust across the r range you tested — no tipping within that range.

- Actionable: strengthening growth mechanisms (policies that increase r) shifts system away from collapse but here is not the limiting factor.

2) A (Allee threshold)
- Pattern: mean_final falls sharply as A increases; collapse probability rises from ~0 to ~1 as A crosses a critical band.
- Implication: there is a clear bifurcation-like sensitivity: raising the effective Allee threshold reduces the basin of attraction of the high state and makes collapse likely. Small uncertainty in A across that band produces large outcome uncertainty.
- Actionable: reducing A (i.e., lowering the threshold needed for recovery — e.g., improving resilience, market connectivity, liquidity backstops) is a powerful lever.

3) c (depletion coefficient)
- Pattern: mean_final decreases and collapse probability jumps to ~1 at modest c; you show a steep transition region.
- Implication: depletion is the dominant destabilizer. A narrow range of c separates safe vs collapse regimes; the system is highly sensitive to increases in depletion.
- Actionable: prioritize interventions that reduce depletion (regulatory limits, buffers, reducing extraction/withdrawal pressures).

4) beta (nonlinear exponent)
- Pattern: mean_final and frac_below show a step change: at low beta you get one regime, at higher beta another (red line jumps to 1 at some beta).
- Interpretation: increasing beta concentrates depletion effects nonlinearly. If beta crosses a threshold, depletion acts more strongly at particular x ranges (depending on c), causing rapid regime change.
- Actionable: examine what mechanisms change the nonlinearity of depletion (e.g., thresholded behaviors, leverage, nonlinear feedbacks). Policies reducing the effective nonlinearity (flatten feedbacks) can stabilize the system.

5) sigma (noise)
- Pattern: mean_final slowly declines as sigma increases; collapse probability rises from near 0 to near 1 for sigma above a critical value.
- Interpretation: stochastic forcing can induce collapse even when deterministic dynamics would maintain a high state — classic noise-induced tipping. There’s a σ_crit where noise makes downward excursions past A common.
- Actionable: reduce volatility (market circuit breakers, liquidity facilities, hedging) to lower collapse risk; recognize that even moderate noise can be dangerous if the deterministic system is near threshold.

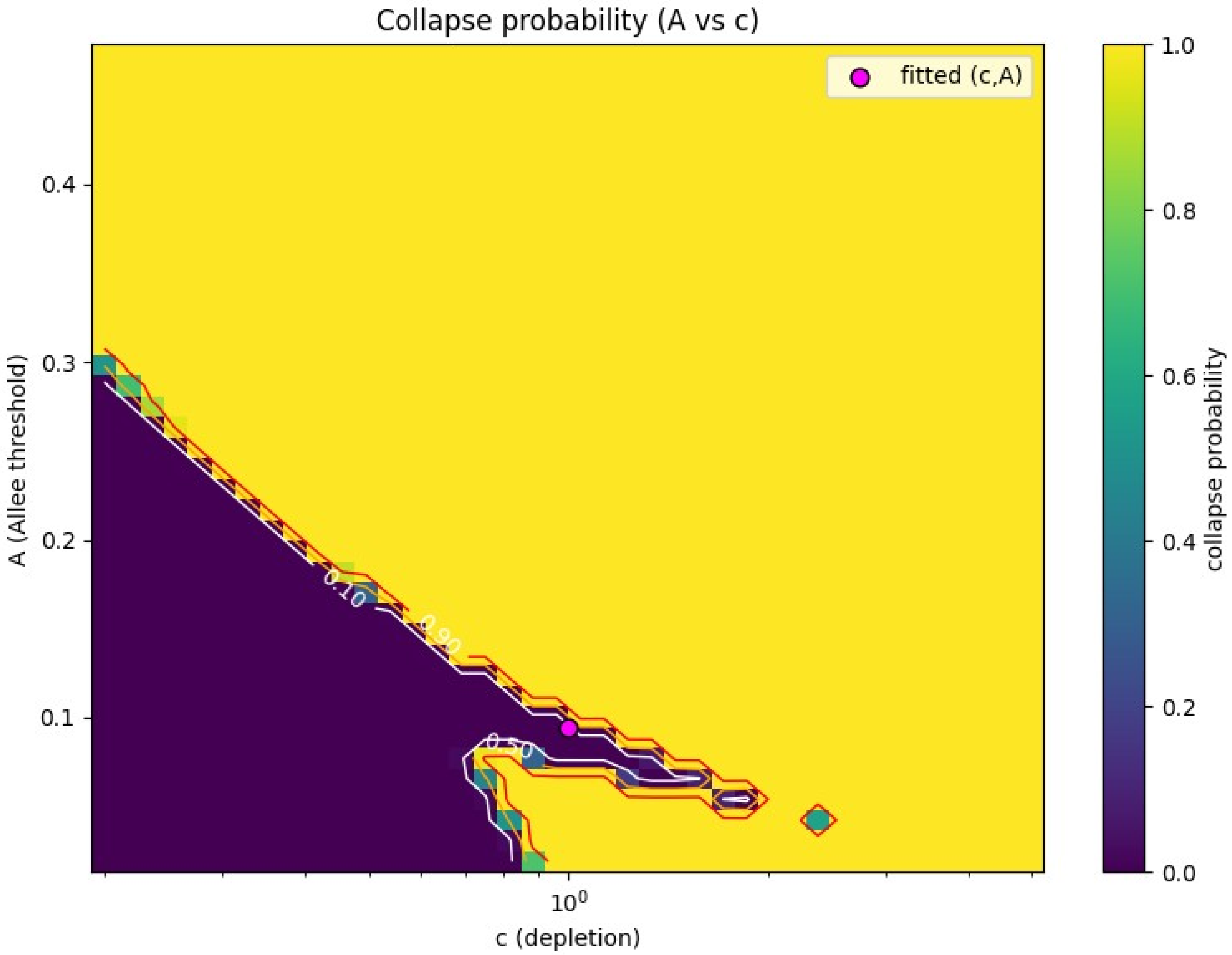


Figure 5: Depletion and Allee threshold heat map.

Reducing depletion (c) and lowering volatility (σ) are the most effective and direct levers to move the system away from the tipping region. Reading the plot:
Axes:

- x-axis = `c` depletion pressure
- y-axis = A survivability threshold

Color:

- purple = survivable (**`frac_below ≈ 0`**)
- yellow = collapse region (**`frac_below ≈ 1`**)

White contours:

- transition probabilities.

Magenta point:

fitted `(c, A)`.

The survivable region forms a **triangular wedge**.

Meaning:

As depletion rises:

- survivability threshold must fall.

Or equivalently:

> the more extraction/depletion pressure a system experiences, the lower the minimum viability requirements must become to avoid collapse.

That is an extraordinarily intuitive systems result.

Translated socially:

If:

- depletion,
- extraction,
- costs,
- instability,
- debt burden,
- ecological strain

rise,

then the system survives only if:

- populations tolerate lower living standards,
- lower resilience,
- lower safety margins.

The fitted point lies:

- just inside, or perhaps slightly beyond, the collapse side of the transition boundary.

That is why, under ensemble simulation the following results returned.

```
collapse probability ≈ 1.0
mean final x ≈ 0.0
```

Meaning:

- stochastic trajectories overwhelmingly decay below the threshold.

The fitted point is very close to the **phase boundary**.

The fitted regime appears close to a sharp transition surface.

Because systems near boundaries:

- can appear stable for long periods,
- yet become highly sensitive to perturbation.

That kind of behaviour is usually seen in:

- climate tipping systems,
- financial crises,
- ecological overshoot,
- and infrastructure fragility.

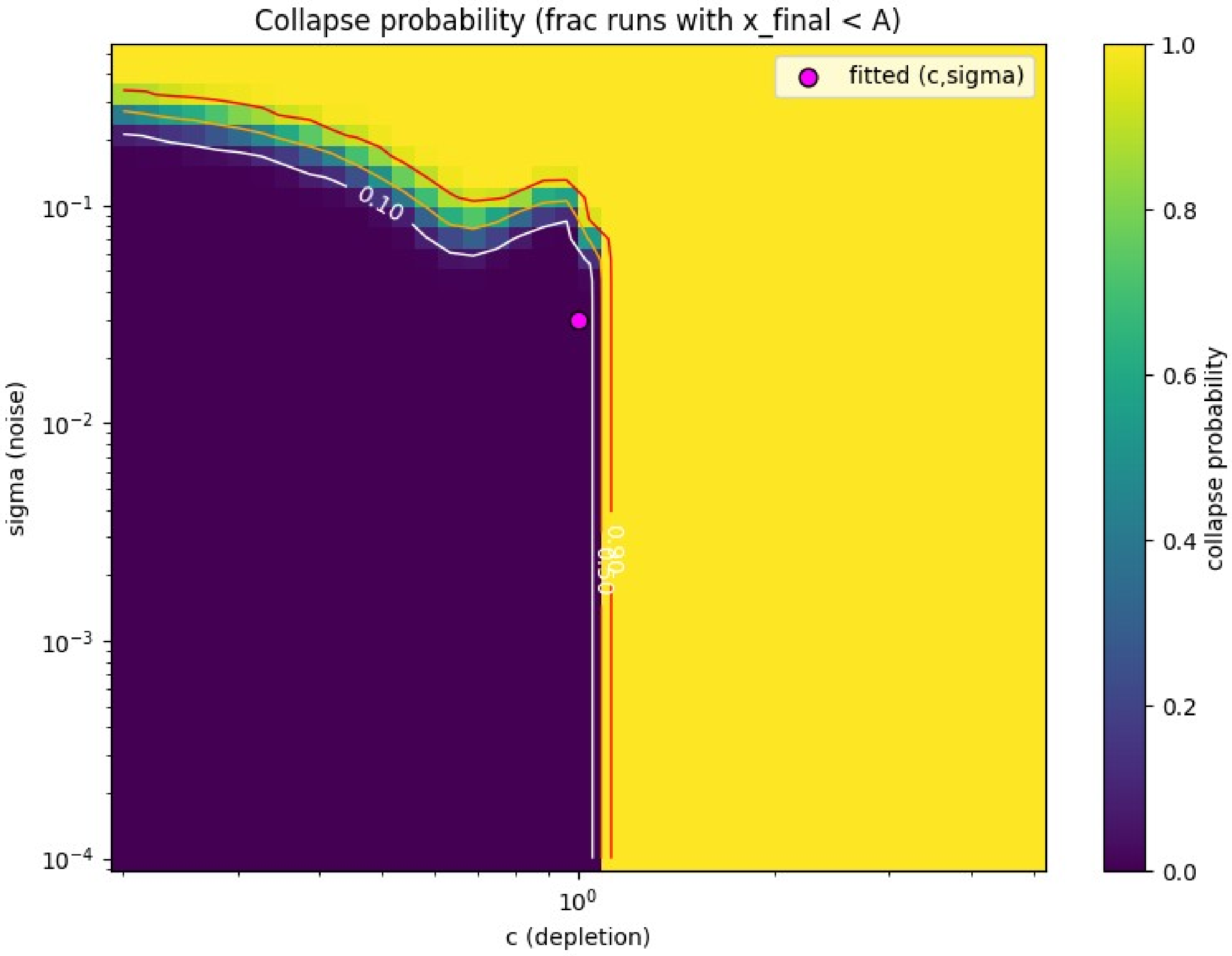


Figure 6: Volatility and depletion heat map

Conclusions

- Lowering c shifts us out of the high-collapse-probability band and raises the ensemble mean final x (reduces deterministic downward pressure).
- Lowering σ reduces noise-induced excursions below the Allee threshold, cutting collapse probability even when deterministic dynamics are marginal.
- If both c and σ remain high, even increases in intrinsic growth r or small changes to A may not prevent collapse — the interaction matters (heatmaps show this).

Recommended priorities

1. Reduce depletion drivers first (policy/intervention on c).
2. Simultaneously reduce volatility (buffers, liquidity backstops, smoothing) to lower σ.
3. If feasible, strengthen resilience (reduce A or increase effective recovery mechanisms) as a structural safeguard.
4. Monitor leading indicators (variance, autocorrelation) for early warnings when the system sits near the contour lines.

It is important to note that phase diagrams do not predict exact events.

Modern systems may remain operational while occupying increasingly narrow resilience margins under rising depletion pressure.

It is important to remember that this data is not a tiny recent sample. It is modeling the real-term evolution of **$100 invested from 1928–2025** across nearly a century of regimes:

Great Depression, WWII, Bretton Woods, oil shocks, neoliberal financialisation, dotcom, GFC, QE, pandemic shock, AI bubble/transition.

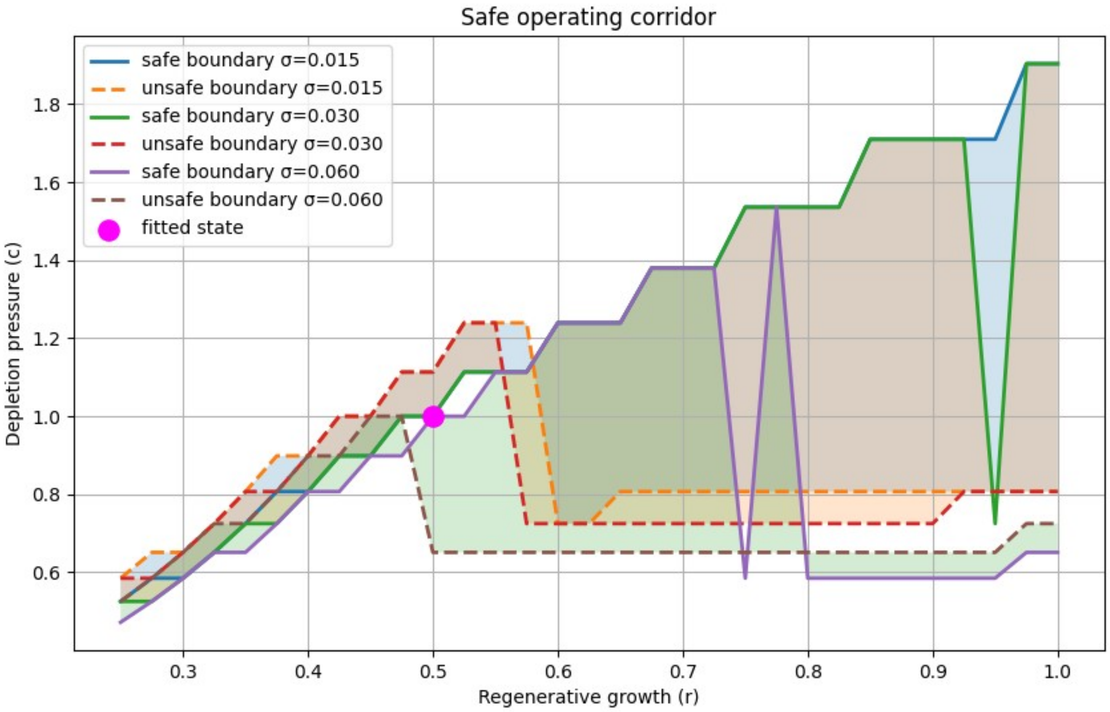
Over a century-long real-return history, the fitted system sits close to a nonlinear depletion boundary. The result should not be read as a deterministic forecast, but as a warning signal: the model’s safe region narrows sharply when depletion rises, and the fitted state appears close to that cliff.

In summary, this analysis points out that there is a resilience-margin warning from a century-scale real-return phase diagram. It is a signal rather than a prediction.

Solution Trade Offs

The heatmaps and plots, and the values of beta (non-linear exponent) suggest a nonlinear resilience envelope.

Optimising an onward path using simulated ensemble statistics produced the following plot. Data is tabled in the Addenda section below.



The fitted point:

`(r ≈ 0.5, c ≈ 1.0)`

sits:

- almost exactly on the transition edge.

This may warrant an early warning signal interpretation.

What the lines mean:

Solid lines:

- maximum safe depletion.

Dashed lines:

- minimum unsafe depletion.

The shaded band:

- transition / metastability corridor.

Meaning:

- small changes can flip outcomes.

That is classic threshold-system behaviour.

Low sigma (0.015)

The system tolerates:

- broader depletion ranges,
- especially at higher `r`.

Meaning:

> lower volatility expands resilience is a reasonable interpretation.

Medium sigma (0.03)

The safe corridor narrows.

The fitted point lies almost exactly on the edge.

This is probably your the most realistic regime.

High sigma (0.06)

The corridor:

- fragments,
- folds,
- and destabilizes.

Those weird purple discontinuities may imply

- disconnected safe pockets,
- attractor fragmentation,
- or numerical bifurcation behaviour.

This is a feature of a nonlinear scenario. The odd rises and falls may indicate

- resonance windows,
- metastable islands,
- or chaotic transitions.

Nonlinear systems examples:

- turbulence,
- climate attractors,
- logistic maps,
- ecological predator-prey systems.

One interpretation is that the system survives only inside bounded combinations of regenerative growth and tolerable depletion.

That resembles:

- ecological carrying capacity,
- sustainability science,
- resilience engineering,
- and adaptive systems theory.

The fitted point visually sits on the ridge, indicating that the resilience margin may be narrow.

Estimated safe operating corridors under varying volatility regimes. Solid lines show the highest depletion pressure compatible with low collapse probability (≤10%), while dashed lines show the onset of unstable regimes. The fitted state (magenta) lies close to a transition boundary, suggesting reduced resilience margins under current conditions.

It is important to note that regenerative growth alone cannot indefinitely compensate for volatility and depletion.

The data suggests progressive erosion of resilience under persistent depletion pressure.

The ensemble spread early in the run suggests:

- temporary adaptability,
- variability in outcomes,
- and short-term resilience differences.

This study is an initial exploration of the sustainability of the underlying adaptive structure. A clear conclusion is that there is a clear difference between price appreciation, and underlying systemic resilience.

Addenda

Historical Returns on Stocks, Bonds and Bills: 1928-2024

Data Used: Multiple data services

Data: Historical Returns for the US

Date: January 2026

Data Source: https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html

Appendix A: Cross-cutting diagnostics: what to look for

- Critical bands: identify parameter ranges where frac_below transitions rapidly from ~0 to ~1 — these are operational tipping windows. Provide ranges (e.g., A in [x1,x2], c in [y1,y2], sigma>z).
- Early-warning indicators: when the system sits near a bifurcation, expect slower recovery from perturbations and rising variance/auto-correlation. Compute autocorrelation and variance in ensemble or data residuals near those parameter values.
- Interaction effects: if both c and sigma moderate collapse, two-parameter heatmaps (c vs sigma, A vs c) will show whether simultaneous moderate changes in both are especially dangerous.
- Time horizon matters: if median_time is very large (inf within T_scan), collapse risk is long-term; if median_time is short, risk is imminent — prioritize based on horizon.

Heatmaps are generated to report collapse probability and median time together to highlight “safe” vs “risky” regions and provide numeric boundaries for critical transitions.

These outcomes can provide input into policy, to target the parameter(s) with largest sensitivity that can be realistically influenced

Appendix B: Depletion safety output from model

Table 1: Depletion safety statistics

Volatility (sigma)	Growth (r)	Max depletion (max_c_to_be_safe)	Min depletion (min_c_to_be_unsafe)
0.015	0.249999974523395	0.525305560880753	0.584803547642573
0.015	0.274999971975735	0.584803547642573	0.651040489200101
0.015	0.299999969428074	0.584803547642573	0.651040489200101
0.015	0.324999966880414	0.651040489200101	0.724779663677696
0.015	0.349999964332753	0.724779663677696	0.806870800810205
0.015	0.374999961785093	0.806870800810205	0.8982598737616
0.015	0.399999959237432	0.806870800810205	0.8982598737616
0.015	0.424999956689772	0.8982598737616	1

0.015	0.449999954142111	0.8982598737616	1
0.015	0.474999951594451	1	1.1132635768448
0.015	0.49999994904679	1	1.1132635768448
0.015	0.52499994649913	1.1132635768448	1.23935579152929
0.015	0.549999943951469	1.1132635768448	1.23935579152929
0.015	0.574999941403809	1.1132635768448	1.23935579152929
0.015	0.599999938856148	1.23935579152929	0.724779663677696
0.015	0.624999936308488	1.23935579152929	0.724779663677696
0.015	0.649999933760827	1.23935579152929	0.806870800810205
0.015	0.674999931213167	1.37972966146122	0.806870800810205
0.015	0.699999928665506	1.37972966146122	0.806870800810205
0.015	0.724999926117846	1.37972966146122	0.806870800810205
0.015	0.749999923570185	1.53600277799718	0.806870800810205
0.015	0.774999921022525	1.53600277799718	0.806870800810205
0.015	0.799999918474864	1.53600277799718	0.806870800810205
0.015	0.824999915927204	1.53600277799718	0.806870800810205
0.015	0.849999913379544	1.7099759466767	0.806870800810205
0.015	0.874999910831883	1.7099759466767	0.806870800810205
0.015	0.899999908284223	1.7099759466767	0.806870800810205
0.015	0.924999905736562	1.7099759466767	0.806870800810205
0.015	0.949999903188902	1.7099759466767	0.806870800810205
0.015	0.974999900641241	1.90365393871588	0.806870800810205
0.015	0.999999898093581	1.90365393871588	0.806870800810205
0.03	0.249999974523395	0.525305560880753	0.584803547642573
0.03	0.274999971975735	0.525305560880753	0.584803547642573
0.03	0.299999969428074	0.584803547642573	0.651040489200101
0.03	0.324999966880414	0.651040489200101	0.724779663677696
0.03	0.349999964332753	0.724779663677696	0.806870800810205
0.03	0.374999961785093	0.724779663677696	0.806870800810205
0.03	0.399999959237432	0.806870800810205	0.8982598737616
0.03	0.424999956689772	0.8982598737616	1
0.03	0.449999954142111	0.8982598737616	1
0.03	0.474999951594451	1	1.1132635768448
0.03	0.49999994904679	1	1.1132635768448
0.03	0.52499994649913	1.1132635768448	1.23935579152929
0.03	0.549999943951469	1.1132635768448	1.23935579152929
0.03	0.574999941403809	1.1132635768448	0.724779663677696
0.03	0.599999938856148	1.23935579152929	0.724779663677696
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0.03	0.724999926117846	1.37972966146122	0.724779663677696
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0.06	0.52499994649913	1	0.651040489200101
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0.06	0.699999928665506	1.37972966146122	0.651040489200101
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